

Asymptotic stability of the sine-Gordon kink outside symmetry

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We consider scalar field theories on the line with Ginzburg-Landau (doublewell) self-interaction potentials. Prime examples include the ϕ^4 model and the sine-Gordon model. These models feature simple examples of topological solitons called kinks. The study of their asymptotic stability leads to a rich class of problems owing to the combination of weak dispersion in one space dimension, low power nonlinearities, and intriguing spectral features of the linearized operators such as threshold resonances or internal modes. We present a perturbative proof of the full asymptotic stability of the sine-Gordon kink outside symmetry under small perturbations in weighted Sobolev norms. The strategy of our proof combines a space-time resonances approach based on the distorted Fourier transform to capture modified scattering effects with modulation techniques to take into account the invariance under Lorentz transformations and under spatial translations. A major difficulty is the slow local decay of the radiation term caused by the threshold resonances of the non-selfadjoint linearized matrix operator around the modulated kink. Our analysis hinges on two remarkable null structures that we uncover in the quadratic nonlinearities of the evolution equation for the radiation term as well as of the modulation equations. The entire framework of our proof, including the systematic development of the distorted Fourier theory, is general and not specific to the sine-Gordon model. This is a joint work with Jonas Lührmann (Texas A&M)

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