

Real and bi-Lipschitz versions of the Theorem of Nobile

Edson Sampaio

UFC (Brazil)

The famous Theorem of Nobile says that a pure dimensional complex analytic set X is analytically smooth if, and only if, its Nash transformation $\eta: \mathcal{N}(X) \rightarrow X$ is an analytic isomorphism. This result was proven in 1975 and since then, as far as the author knows, no answer has been given to the real case, even more so when one only asks for C^k smoothness. In this paper, we prove the real version of the Theorem of Nobile asking only C^k smoothness, i.e., we prove that for a pure dimensional real analytic set X the following statements are equivalent: (1) X is a real analytic (resp. $C^{k+1,1}$) submanifold; (2) the mapping $\eta: \mathcal{N}(X) \rightarrow X$ is a real analytic (resp. $C^{k,1}$) diffeomorphism; (3) the mapping $\eta: \mathcal{N}(X) \rightarrow X$ is a C^∞ (resp. $C^{k,1}$) diffeomorphism; (4) X is a C^∞ (resp. $C^{k+1,1}$) submanifold. In this paper, we also prove the bi-Lipschitz version of the Theorem of Nobile. More precisely, we prove that X is analytically smooth if and only if its Nash transformation $\eta: \mathcal{N}(X) \rightarrow X$ is a homeomorphism that locally bi-Lipschitz.

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