

Existence of positive solutions for a semipositone $p(\cdot)$ -Laplacian problem

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In this presentation, we present the results obtained in collaboration with Lucas A. Vallejos from Universidad Nacional de Córdoba, see [3]. These results are the only ones so far in the literature addressing a semipositone problem for the $p(\cdot)$ -Laplacian. Here, we establish the existence of a positive weak solution for a semipositone $p(\cdot)$ -Laplacian problem, considering specifically the problem

$$(P) \quad \begin{cases} -\Delta_{p(\cdot)} u = f(u) - \lambda & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases},$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$ is a smooth bounded domain, f is a continuous function with subcritical growth, $\lambda > 0$ and $\Delta_{p(\cdot)} u = \operatorname{div}(|\nabla u|^{p(\cdot)-2} \nabla u)$. By assuming an Ambrosetti-Rabinowitz type condition and employing Mountain Pass arguments, comparison principles, and regularity results, we prove the existence of a positive weak solution for sufficiently small λ . Additionally, we will discuss relevant background results, including those obtained in [1], where the authors study the problem (P) for the Φ -Laplacian operator. The ideas from [1] were adapted to derive the results shown in this presentation. Furthermore, we present the findings from [2] highlighting the challenges in adapting them to the case of the $p(\cdot)$ -Laplacian.

References

- [1] Alves, C., de Holanda, A. and Santos, J. *Existence of positive solutions for a class of semipositone quasilinear problems through Orlicz-Sobolev space*. Proc. Amer. Math. Soc. 147.1: 285-299. (2019).

- [2] Castro, A., de Figueredo, D. G., and Lopera, E. *Existence of positive solutions for a semipositone p -Laplacian problem*. Proc. Roy. Soc. Edinburgh Sect. A, 146A (2016) 475-482.
- [3] Vallejos L. A. and Vidal R. E. *Existence of positive solutions for a semipositone $p(\cdot)$ -Laplacian problem*. J. Math. Anal. Appl. 547 (2025) 129333.

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