

Existence and multiplicity of solutions for nonlocal boundary value problems of Schrödinger type

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Problems involving the fractional p -Laplacian have been an object of intensive research in the last years in many branches of science such as in phase transition phenomena, population dynamics, and game theory. An interesting example is the use this operator to model a random walk of a particle with possibly long jumps.

In this talk, we are concerned with the existence and multiplicity of solutions for the following problem

$$\begin{cases} -(\Delta)_p^s u + V(x)|u|^{p-2}u = \lambda f(u), & x \in \Omega; \\ u = 0, & x \in \mathbb{R}^N \setminus \Omega, \end{cases} \quad (1)$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded set with Lipschitz boundary $\partial\Omega$, $N \geq 2$, $V \in L^\infty(\mathbb{R}^N)$, and $(-\Delta)_p^s$ denotes the fractional p -Laplacian with $s \in (0, 1)$, $1 < p$, $sp < N$, $\lambda > 0$, and $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function.

The work presented in this talk was motivated by the results obtained by Castro, de Figueiredo and Lopera, in [Existence of positive solutions for a semipositone p -Laplacian problem.(2016) Proc. Roy. Soc. Edinburgh Sect. A.] for the case of the p -Laplacian operator and by Lopera, López and Vidal, in [Existence of positive solutions for a parameter fractional p -Laplacian problem with semipositone nonlinearity.(2023) J. Math. Anal. Appl.] for the fractional p -Laplacian. In those articles, the authors proved the existence of a positive solution for problems like (1) when the potential $V \equiv 0$. In both cases the existence was obtained by using variational methods. They also proved that the solution was positive by using some new regularity results and Hopf's Lemma.

The primary goal of this presentation is to extend the aforementioned result

of Lopera, López and Vidal by proving the existence of at least two solutions for problem (1). We use a variant of the mountain-pass theorem and infinite-dimensional Morse theory to obtain the existence of a second solution for both cases where $f(0) \neq 0$ and $f(0) = 0$, respectively.

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