Uniform estimates for elliptic equations with Caratheodory nonlinearities at the interior and on the boundary

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Let us consider the following elliptic problem

$$\begin{cases} -\Delta u + u &= f(x,u), \qquad x \in \Omega\,, \\ \frac{\partial u}{\partial \eta} &= f_B(x,u), \qquad x \in \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$, $(N > 2)$, is an open, connected, bounded domain with C^2 boundary, $\partial/\partial\eta = \eta \cdot \boldsymbol{n}$ is the (unit) outer normal derivative, and the functions $f : \Omega \times \mathbb{R} \to \mathbb{R}$, and $f_B : \partial\Omega \times \mathbb{R} \to \mathbb{R}$, are both slightly subcritical Carathéodory functions, in other words satisfies:

$$|f(x,s)| \le |a(x)|\,\tilde{f}(s) \qquad \text{and} \qquad |f_B(x,s)| \le |a_B(x)|\,\tilde{f}_B(s)$$

with $a \in L^r(\Omega)$, for $r > \frac{N}{2}$, $a_B \in L^r_B(\partial\Omega)$, for $r_B > N-1$ and $\tilde{f}, \tilde{f}_B : [0,+\infty) \to [0,+\infty)$ are continuous and also

$$\lim_{|s|\to\infty} \frac{\tilde{f}(s)}{|s|^{2^*_{N/r}-1}} = 0 \qquad \text{and} \qquad \lim_{|s|\to\infty} \frac{\tilde{f}_B(s)}{|s|^{2_{*,N/r_B}-1}} = 0,$$

where

$$2^*_{N/r} := 2^*\Big(1 - \frac{1}{r}\Big) \quad \text{and} \quad 2_{*,N/r_B} := 2_*\Big(1 - \frac{1}{r_B}\Big)$$

and $2^* := \frac{2N}{N-2}$ and $2_* := \frac{2(N-1)}{N-2}$ will denote the critical Sobolev exponent and the critical exponent in the sense of the trace, respectively.

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Through a De Giorgi-Nash-Moser iteration scheme, it is known that weak solutions to (1) with critical growth are in $L^{\infty}(\Omega)$.

Our contribution is to provide an explicit $L^{\infty}(\Omega)$-estimate of weak solutions with slightly subcritical growth, in terms of powers of their $H^{1}(\Omega)$-norms. Our method combines elliptic regularity of weak solutions with Gagliardo–Nirenberg interpolation inequality.

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