

The Cox Ring of an Embedded Variety

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Let X be a normal subvariety of a smooth Mori dream space Z . The inclusion $X \subseteq Z$ induces natural homomorphisms at the level of divisor class groups and Cox rings. A central problem is to explicitly describe the Cox ring of X starting from its subring R , obtained as the image of the Cox ring of Z under the pullback map. Such a description is crucial for understanding the geometry and birational properties of X . When X is an ample hypersurface and the induced map on divisor class groups is an isomorphism, previous results by Hausen (2008, "Cox Rings and Combinatorics. II") and Artebani and Laface (2012, "Hypersurfaces in Mori Dream Spaces") show that a Lefschetz-type theorem for Cox rings holds under the hypothesis that the irrelevant locus of the ambient variety Z contains no codimension-two components. In this work, we remove this hypothesis and describe the Cox ring of X as the intersection of finitely many localizations of R . As an application, we study the Cox rings of Calabi–Yau hypersurfaces in toric Fano varieties, focusing on the relationship between the effective and moving cones of the ambient space and those of the hypersurface, which may fail to be polyhedral, as shown by Ottem, for example, for products of projective spaces. This is joint work with Cristóbal Herrera and Luca Ugaglia.

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