Annular Seifert surfaces, plumbing-uniqueness and a non-semisimple quantum link polynomial

Daniel López Neumann

Universidad de Concepción

Let A(n) be an unknotted annulus with n full twists. We will say that a Seifert surface is annular of type (i,j,k,. . .) if it is obtained by plumbing A(i), A(j), A(k),. . . in some way. Is it possible for a knot to bound an annular surface of type (2,2,1,1) and a surface of type (4,1,1,1)? The top coefficient of the Alexander polynomial and the top group of knot Floer homology are plumbing-multiplicative, but they do not obstruct the preceding situation to happen. We answer the above question in the negative, in fact, we prove a plumbing-uniqueness result for links that bound annular surfaces. We do this by showing that a certain quantum link polynomial, namely the top coefficient of the 2-variable Links-Gould invariant, is plumbing-multiplicative. This marks a sharp difference between this lesser-known "non-semisimple" quantum invariant from the widely studied (semisimple) Jones and HOMFLY quantum link invariants.

In Special Session 31 on Structures of Submanifolds in Low Dimensional Topology at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS31-T11