

Asymptotic Expansions in Habiro Rings

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Over the complex numbers, differential equations with holomorphic coefficients have holomorphic solutions. The solutions of a $\hbar$ -family of differential equations with holomorphic coefficients, are holomorphic over the complex numbers, but they have an essential singularity at infinity. Through the WKB method, topological recursion, gives a formalism for computing asymptotic expansions of solutions to $\hbar$ -deformed differential equation at the essential singularity over the complex numbers. In a recent work, Garoufalidis, Scholze, Wheeler and Zagier investigate asymptotic expansions in Habiro rings. The authors study solutions to the q -difference equation and provide asymptotics expansions of the t -deformed Nahm sums at all roots of unity, for a positive definite square matrix. In the particular example, of a rank one matrix, the asymptotics expansions obtained recover as the functional equation, the spectral curve of the Catalan numbers. I will talk about the differences and similitudes of these two asymptotic expansions the first one over the complex numbers and the later in Habiro rings.

In Special Session 32 on Vector Bundles on Varieties and Quantization at the Mathematical Congress of the Americas 2025.