The Volterra function in the Lyapunov method

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The Volterra function became quite ubiquitous in mathematical epidemiology after being rediscovered by Korobeinikov and Wake in [1]. Its usage met with such a highly successful track record that a meta theorem seems to have emerged in the mathematical epidemiology community: Given a epidemiological model with a single Endemic Equilibrium when R0>1, then its global stability can be proved by an appropriate choice of coefficients of the underlying Volterra function — this is usually the hardest part in proving that a model satisfy the sharp threshold property [2]. I'll briefly review some of the successful applications of the Volterra function in epidemiology and show how it fail in an ODE model for malaria [3]. References [1] A Korobeinikov and GC Wake (2002), Lyapunov functions and global stability for SIR, SIRS and and SIS epidemiological models, Appl Math Letters 15:955–960.[2] Z Shuai and PV Den Driessche (2013), Global stability of infectious disease models using Lyapunov functions, SIAM J Appl Math 73(4):1513–1532.[3] A Iggidr and MO Souza (2023), On the limits of the Volterra function in the Lyapunov method: The Anderson-May-Gupta as a cautionary example, J Math Anal Appl 517(1):126465.

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