

The discriminant of imaginary quadratic fields and Kontsevich's tropical approach to disproving the Hodge conjecture for Weil type abelian varieties

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Several years ago, Kontsevich proposed an idea to disprove the Hodge conjecture for Weil type cycles on abelian varieties. The broad outlines of this idea are recorded for abelian fourfolds in I.Zharkov's 2020 ArXiv preprint "Tropical Abelian varieties, Weil classes and the Hodge Conjecture." In the meantime, Markman has proved the Hodge conjecture for all abelian fourfolds and for all abelian six folds of discriminant -1 . I'll explain why Kontsevich's method can only work for Weil type $2n$ -folds of discriminant 1 , when n is even, and -1 when n is odd. In particular, given Markman's most recent results, this shows that the Kontsevich's method will not work until the dimension of the abelian variety is at least 8 .

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