

Tropical theta functions for cluster varieties

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A fundamental fact in toric geometry says that the discrete valuation of a linear combination of monomials along a toric boundary divisor is equal to (never greater than) the minimum of the valuations of the terms. We call this valuative independence. I will discuss joint work with M.-W. Cheung, T. Magee, and G. Muller in which we show that theta functions constructed from a positive scattering diagram (e.g., those on cluster varieties) also satisfy valuative independence. Applications include finding theta function bases for global sections of line bundles, linear independence of theta functions with specialized coefficients, and a general gluing lemma for theta functions from moduli of local systems on marked surfaces. We then prove theta reciprocity, a symmetry property between valuations and theta functions on mirror pairs: roughly, $\text{valu}(\theta u) = \text{valu}(\theta v)$.

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