

Propagating terrace in semi-discrete models of Incompressible Porous Medium Equation (IPM)

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In the talk we will discuss gravitational fingering phenomenon - the unstable displacement of miscible liquids in porous media with the speed determined by Darcy's law. Laboratory and numerical experiments show the linear growth of the mixing zone, and we are interested in determining the exact speed of propagation of fingers. We consider two models: incompressible porous medium equation (IPM) and Transverse Flow Equilibrium (TFE). The existing theoretical upper bounds for the growth rate of the mixing zone (see e.g. Otto-Menon'2005) are higher than the observed speed from the numerical simulations. We believe that one of the possible mechanisms of slowing down the fingers' growth is due to convection in the transversal direction, and explain it by introducing a semi-discrete model of IPM and TFE. The model consists of a system of advection-reaction-diffusion equations on concentration, velocity and pressure in several vertical tubes (real lines) and interflow between them. In the simplest setting of two tubes we show the structure of gravitational fingers - the profile of propagation is characterized by two consecutive travelling waves which we call a terrace. We prove the existence of such a propagating terrace for the parameters corresponding to small distances between the tubes. The main tool in the proof is the reduction of IPM model to TFE model using geometric singular perturbation theory. The talk is based on joint work with S. Tikhomirov and Ya. Efendiev (arXiv:2401.05981, SIMA, 2025).

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