Stable singularity formation for the inviscid Primitive equations

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The primitive equations (PEs) model large-scale dynamics of the oceans and the atmosphere. While it is by now well known that the three-dimensional viscous PEs are globally well posed in Sobolev spaces, and that there are solutions to the inviscid PEs (also called the hydrostatic Euler equations) that develop singularities in finite time, the qualitative description of the blowup still remains undiscovered. In this talk, we provide a full description of two blow-up mechanisms, for a reduced PDE that is satisfied by a class of particular solutions to the PEs. In the first one a shock forms, and pressure effects are subleading, but in a critical way: they localize the singularity closer and closer to the boundary near the blow-up time (with a logarithmic-in-time law). This first mechanism involves a smooth blow-up profile and is stable among smooth enough solutions. In the second one the pressure effects are fully negligible; this dynamics involves a two-parameter family of non-smooth profiles, and is stable only by smoother perturbations. At the end, I will briefly discuss how the results extend when the system is coupled with a density equation. These are joint works with C. Collot, Q. Lin, Titi and Qian.

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