

On the inviscid instability of the 2D Taylor–Green vortex

Bartosz Protas

McMaster University, Canada

We consider Euler flows on a 2D periodic domain and are interested in the stability, both linear and nonlinear, of a simple equilibrium given by the 2D Taylor–Green vortex. As the first main result, numerical evidence is provided for the fact that such flows possess unstable eigenvalues embedded in the band of the essential spectrum of the linearized operator. However, the unstable eigenfunction is a distribution unbounded at the hyperbolic stagnation points of the base flow and its regularity is consistent with a theorem of Lin (2004). This eigenfunction gives rise to an exponential transient growth with the rate given by the real part of the eigenvalue followed by passage to a nonlinear instability. As the second main result, we illustrate a fundamentally different, non-modal, growth mechanism involving a continuous family of uncorrelated functions, instead of an eigenfunction of the linearized operator. Constructed by solving a suitable PDE optimization problem, the resulting flows saturate the known estimates on the growth of the semigroup related to the essential spectrum of the linearized Euler operator as the numerical resolution is refined. These findings are contrasted with the results of earlier studies of a similar problem conducted in a slightly viscous setting where only the modal growth of instabilities was observed. This highlights the special stability properties of equilibria in inviscid flows.[Joint work with Xinyu Zhao (NJIT) and Roman Shvydkoy (University of Illinois at Chicago)]

In Special Session 36 on New Developments in Mathematical Fluid Dynamics at the Mathematical Congress of the Americas 2025.