On oscillation of the second order ODE and the 1st order delay equation

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"We discuss the oscillation theory of the second order differential equation

$$x''(t) + p(t)x(t) = 0, \tag{1}$$

where

$$p(t) \geq 0.$$

The oscillation of (1) is known to be equivalent to the oscillation ofthe first order delay equation

$$y'(t) + (\frac{1}{e} + \frac{p(t)}{2e})y(t-1) = 0, \tag{2}$$

which is at the critical state. This result is based on transformations typically applied to neutral equations. A recent, independent, center-manifold approachconstructed a two-dimensional ODE system, whose oscillation is alsoequivalent to the oscillation of (2), even when $p(t)$ is allowed totake both positive and negative values. What is more, we prove thatthe two-dimensional center manifold system coincides with (1), up tosome error terms."

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