Hyperbolic problems and related models in mineral processing and wastewater treatment

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This contribution is a survey on recent advances on the formulation, mathematical analysis, and numerical solution for models of solid-liquid or liquid gas multiphase flows in applications of mineral processing and wastewater treatment. These include batch and continuous sedimentation, countercurrent decantation, column flotation, and reactive settling. The models are based on balance equations of continuum mechanics followed by a dimensional analysis and further simplifications, and can in the widest sense be formulated as convection-diffusion-reaction partial differential equations. Typically, the sought unknowns are the volume fractions of components of a disperse phase that may either segregate and form areas of different composition (such as particles in a polydisperse suspensions that differ in size or density) or undergo reactions with other components (as occurs in reactive settling involving particles of biomass and various nutrients dissolved in the liquid). A characteristic property of most of the models developed is the reduction to one space dimension (aligned with gravity) and to a finite domain, as typical for unit operations. The latter restriction calls for the expression of boundary conditions by discontinuities of the governing flux, as well as the description of inlets through singular source terms. Some specific applications, and their respective main mathematical challenges that will be addressed include (1) batch and continuous sedimentation of flocculated suspensions in clarifier-thickeners, which are described by a conservation law with discontinuous flux and strongly degenerating diffusion term; (2) polydisperse suspensions modelled first-order systems of conservation laws of arbitrary size for which results of hyperbolicity analysis have recently led to the formulation of invariant-region-preserving high-order WENO schemes; (3) models of reactive settling in wastewater treatment in secondary settling

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tanks (SSTs) and sequencing batch reactors (SBRs) that incorporate reaction terms and in the case of SBRs, call for incorporating moving boundaries; (4) models of froth flotation in columns that involve bubbles and particles and that are defined by systems of conservation laws with discontinuous flux, degenerate diffusion and triangular structure; and (5) settling with convection in an inclined channel described by a coupled flow-transport model. The speaker is partially funded by ANID (Chile) through projects Fondecyt 1210610 and 1250676, Anillo ANID/ACT210030, Centro de Modelamiento Matemático (CMM; BASAL project FB210005), and CRHIAM, projects ANID/Fondap/15130015 and ANID/Fondap/1523A0001.

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