

A geometric approach to Ramsey numbers

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The r -colour Ramsey number $R_r(k)$ is the minimum $n \in \mathbb{N}$ such that every r -colouring of the edges of the complete graph K_n on n vertices contains a monochromatic copy of K_k . We prove, for each fixed $r \geq 2$, that

$$R_r(k) \leq e^{-\delta k} r^{rk}$$

for some constant $\delta = \delta(r) > 0$ and all sufficiently large $k \in \mathbb{N}$. At the heart of our approach is a geometric lemma about the inner product of pairs coupled random vectors.

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