High-dimensional asymptotics of the lr Grothendieck problem and r to p norms of Levy matrices

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Given an n × n symmetric matrix A, the lr Grothendieck problem considers the maximum of the quadratic form x' A x over vectors x in the unit lr sphere. Both this quantity and the r to p operator norm of the matrix A arise in a variety of fields including asymptotic convex geometry, theoretical computer science and statistical mechanics, and are known to be hard to compute or even efficiently approximate in various parameter regimes. In this talk, we identify the high-dimensional asymptotics of these quantities (suitably renormalized) for symmetric random matrices with independent and identically distributed heavy-tailed upper-triangular entries whose index lies in a suitable parameter regime. Our limit theorems complement classical results in the well studied spectral case when both r and p are equal to 2, as well as recent work on asymptotics for operator norms of Gaussian and non-negative random matrices. The analysis uses different techniques that are not reliant on spectral methods, and yields new scaling limits that do not arise in the spectral setting. As a corollary of our results we also characterize the limiting ground state of the Levy spin glass model. We will also mention several open problems. This is based on joint work with Xiaoyu Xie.

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