Analysis of the Efficiency of SPC Codes through Bipartite Graphs

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The single parity-check (SPC) code is a widely used error detection coded due to its simplicity and ease of implementation [5]. It works by appending a single bit to an information sequence, making the total number of ones in the sequence even. When two SPC codes are combined, they form an SPC product code with a minimum distance of 4, allowing it to recover from up to three erasures on the erasure channel [3]. In some cases, it can even correct patterns with up to $2n-1$ erasures. An SPC codeword can be represented as an erasure pattern in a grid of size $n \times n$, where the key information is the position of the erasures.

In previous studies [5], it was shown that erasure patterns can be categorized into correctable and uncorrectable patterns. For example, patterns that contain certain subpatterns that cannot be corrected are uncorrectable. After that, in [1], the authors used combinatorics to determine the number of uncorrectable erasure patterns for different numbers of erasures. These patterns were represented as bipartite graphs, where the edges between nodes represented the erasures in the pattern [1]. A key insight was that uncorrectable patterns are represented by graphs with cycles.

This leads to the challenge of counting bipartite graphs with cycles. These graphs can also be represented using binary matrices, where a “1” indicates a connection (edge) between two nodes. The counting problem is thus reduced to determining binary matrices with certain properties. The authors of [2] analysed the bi-adjacency matrix [4] of the bipartite graphs to provide upper and lower bounds on the number of uncorrectable erasure patterns. The exact number of correctable or uncorrectable patterns remains an open question, with limited progress made so far.

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In this work, it is shown that each erasure pattern can be represented by a binary sequence, where each bit corresponds to a position in the erasure pattern: "1" if there is an erasure at that position, and "0" if there is not, that is, any pattern can be represented as a sequence of ones and zeros, matching the positions of the erasures in the pattern.

We also study the parity-check matrix of the SPC product code and how certain columns of this matrix form linearly dependent sets, which correspond to uncorrectable patterns. By examining these relationships, the study aims to find bounds on the number of uncorrectable erasure patterns and evaluate the performance of the SPC code.

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