

Fields containning the algebraic closure of the field of series and Abhyankar-Moh Semigroups

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Over an algebraically closed field of characteristic zero, the algebraic closure of the field of series in one variable is the field of Puiseux series. When working with n variables, the natural extension of Newton's Puiseux theorem works only for quasi-ordinary singularities [1]. For each continuous order in $\mathbb{Q}_n$, in [3, 4] we construct an algebraically closed field that contains the algebraic closure of the field of series in n variables. Manfred Buchacher [5] has implemented an algorithm using Mathematica that computes the first terms of the roots in these fields. In the talk I will explain the construction of these fields and how they lead to a natural generalization of the concept of Abhyankar-Moh Semigroups [2]. Joint work with: Annel Ayala Velasco and Giovanna Ilardi¹, Instituto de Matemáticas, Universidad Nacional Autónoma de Mexico, Oaxaca, Mexico.

References

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*Partially supported by UNAM PAPIIT-IN113323, e-mail: fuen@im.unam.mx

In Special Session 41 on Algebraic Geometry: the Numerical, the Random and the Tropical at the Mathematical Congress of the Americas 2025.