On the topology of tropical varieties

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Mikhalkin's construction of a non-planar tropical cubic curve of genus 1 in R3 represented a major breakthrough in the study of combinatorial tropical varieties. It provided the first known example of a non-realizable tropical variety, where the obstruction is topological in nature. More recently, it has been shown that the upper bound on the top Betti number of combinatorial tropical varieties—given fixed dimension and degree—also depends on the codimension, in contrast to the classical complex case. Specifically, for any three positive integers d, m, and kkk, there exist tropical varieties of degree ddd, dimension mmm, and codimension kkk whose top Betti number equals kkk times the classical upper bound for complex varieties of the same degree and dimension. These examples are constructed using the method of Floor composition. In low dimensions and degrees, the resulting constructions are maximal. This is a joint work Benoît Bertrand and Erwan Brugallé.

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