

Robust Polyhedral Homotopy Continuation

Jan Verschelde

University of Illinois at Chicago

To solve a given system of polynomial equations, a homotopy method connects the given system to a start system, a system with known solutions. The homotopy defines solution paths, from the known solutions to the solutions of the given system. Those paths are tracked by continuation methods. Robust path trackers [Telen, Van Barel, Verschelde, SISC 2020] adjust the step size in the continuation method based on the curvature of the paths and the proximity to the nearest singularity. The location of nearest singularity is computed via the application of the theorem of Fabry using the Taylor series expansions of the solution paths in the continuation parameter. Sparse polynomial systems, which have equations with few monomials appearing with nonzero coefficient, are best solved via polyhedral homotopies. The powers of the continuation parameter in a polyhedral homotopy are typically real numbers. Instead of Taylor series, we then have to consider series developments with real exponents and apply algorithms suggested by tropical geometry. This is joint work with Katherine Kruzan (UIC).

In Special Session 41 on Algebraic Geometry: the Numerical, the Random and the Tropical at the Mathematical Congress of the Americas 2025.