

Higher Order Arithmetic-Geometric Inequalities

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If $\{\alpha_k\} \subset \mathbb{R}^d$ consists of a simplex and a single interior point, and if you impose the condition that $p(x) = \sum c_k x^{\alpha_k}$ vanishes to the second order at $\underline{1} = (1, \dots, 1)$, then the resulting polynomial is, up to a multiple, a version of the arithmetic-geometric inequality for the monomials $\{x^{\alpha_k}\}$. In this talk, we explore geometric conditions on $\{\alpha_k\}$ so that imposing a higher even-order vanishing at $\underline{1}$ leads to an inequality, and present a few preliminary results.

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