

# Rational Eigenfunctions of the Hecke Operator

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We consider the action of the Hecke Operators  $U_n$ , which are sometimes called Cartier Operators, on the space of rational functions over  $\mathbb{C}$ . Our main goal is to give a classification of the eigenfunctions of  $U_n$  based on certain number-theoretic directed graphs, called Zolotarev Graphs. These graphs extend the well-known Zolotarev Permutations that were classically used to prove quadratic reciprocity. Moreover, the Zolotarev Graphs are naturally induced by certain subspaces of the rational functions, and they provide crucial information about the dimension of each eigenspace and the kernel of  $U_n$ . The Zolotarev Graphs are furthermore directly related to another classical topic, namely unitary divisors, studied by Cohen. Finally, we study the vector space of simultaneous eigenfunctions, which are eigenfunctions of all of the operators  $U_n$ , and we give explicit bases for it. Finally, we prove that the classical Artin Conjecture on primitive roots is equivalent to a new conjecture in this context, that infinitely many of these eigenspaces have dimension 1.

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