

Triangular Numbers, Catalan Numbers, and Diagonal Lattice Paths

Malachi Robinson

University of Illinois at Urbana-Champaign

We define a sequence of matrices related to resultants of modified Fibonacci polynomials. These matrices always contain a column containing a simple transformation of the Catalan numbers. To show this, we prove a combinatorial identity that expresses the triangular numbers in terms of the Catalan numbers, and several related results. The proof involves the enumeration of lattice paths that remain strictly above the line $y = x$ with diagonal $D = (1,1)$ moves allowed.

In Special Session 42 on Combinatorial Number Theory in the Integer Lattice at the Mathematical Congress of the Americas 2025.