

Lattice angles and quadratic forms

Lenny Fukshansky

Claremont McKenna College

What are the possible angles between two vectors in the integer lattice $\mathbb{Z}^n$? If we fix one such possible angle and one integer vector x , is there always another integer vector y that makes this angle with x ? Assuming that x makes a given angle with some vector, how can we find the shortest such vector y ? What if we designate a forbidden set of vectors, what is the shortest y making a given angle with x outside of this forbidden set? It turns out that all of these questions can be reformulated in terms of a search for integer zeros of integral quadratic forms, a rich arithmetic theory. I will discuss this research direction and show results by several authors, including my recent joint work with Sehun Jeong.

In Special Session 42 on Combinatorial Number Theory in the Integer Lattice at the Mathematical Congress of the Americas 2025.