

On the orbital stability of solitary waves for the fourth order nonlinear Schrödinger equation

Fabio Natali

Universidade Estadual de Maringa, Brazil

In this talk, we present new results regarding the orbital stability of solitary standing waves for the general fourth-order Schrödinger equation with mixed dispersion. The existence of solitary waves can be determined both as minimizers of a constrained complex functional and by using a numerical approach. In addition, for specific values of the frequency associated with the standing wave, one obtains explicit solutions with a hyperbolic secant profile. Despite these explicit solutions being minimizers of the constrained functional, they cannot be seen as a smooth curve of solitary waves, and this fact prevents their determination of stability using classical approaches in the current literature. To overcome this difficulty, we employ a numerical approach to construct a smooth curve of solitary waves. The existence of a smooth curve is useful for showing the existence of a threshold power $\alpha_0 \approx 4.8$ of the nonlinear term such that if $\alpha \in (0, \alpha_0)$, the explicit solitary wave is stable, and if $\alpha > \alpha_0$, the wave is unstable. An important feature of our work, caused by the presence of the mixed dispersion term, concerns the fact that the threshold value $\alpha_0 \approx 4.8$ is not the same as that established for proving the existence of global solutions in the energy space, as is well known for the classical nonlinear Schrödinger equation.

In Special Session 43 on Harmonic Analysis and Partial Differential Equations at the Mathematical Congress of the Americas 2025.