Session 44 - On the solvability of the Dirichlet problem for homogeneous second order systems in the plane

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The crowning result of the theory of elliptic boundary value problems (BVP's) in smooth domains (formulated in Sobolev-Besov spaces) is the fact that Fredholm solvability is equivalent to the Shapiro-Lopatinskii condition being satisfied. This reduces the decidedly complex task of determining whether such BVP's are solvable uniquely, modulo finite dimensional spaces, to the much more mundane job of checking if a certain ODE does not have nontrivial solutions which vanish at infinity. The proof of this fundamental theorem relies on constructing a parametrix for the solution operator via flattening the boundary and using pseudo-differential operator techniques. Since such an approach makes essential use of the smoothness of the underlying domain, it has been unclear whether the Shapiro-Lopatinskii condition retains any significance in the non-smooth setting. In this talk I will show that this basic open question has a positive answer for the Dirichlet problem for second order, homogeneous, constant real coefficient, 2x2 systems in the plane. The theory presented is worked out in the setting of chord-arc domains and allows the treatment of a large variety of spaces of boundary data, including the classical scale of Lebesgue spaces.

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