Hardy Spaces for the Lamé system in the unit ball in $\mathbb{R}^3$

Emilio Marmolejo-Olea

Instituto de Matematicas Unidad Cuernavaca, UNAM MEXICO

We will study the elastic version of the classical Hardy spaces of harmonic functions in the unit ball $\mathbb{B}$ in $\mathbb{R}^3$. Namely for $1 < p < \infty$ consider

$$h_e^p(\mathbb{B}) = \{u : \mathbb{B} \to \mathbb{R}^3 \mid \Delta^* u = 0 \text{ and } sup_{0 \le r < 1} \int_{\mathbb{S}} |u(rw)|^p d\sigma(w) < \infty\},$$

where $\Delta^* = \mu\Delta + (\lambda + \mu)\nabla div$ is the Lamé operator with constants λ and μ, $\mu > 0$, $\lambda + 2\mu > 0$ and $\mathbb{S}$ is the unit sphere in $\mathbb{R}^3$. We also consider the maximal elastic version of the Hardy spaces $H_e^p(\mathbb{B})$.
First, following Kupradze, we recall the elastic Poisson kernel P_e. From this we will prove in a standard way some classical results, like Fatou Theorem and $h_e^p(\mathbb{B})$ being isomorphic to $L^p(\mathbb{S}, \mathbb{R}^3)$. In particular, it follows that the boundary values do not depend on the Lamé constants. The novel part is that the space $h_e^p\mathbb{B})$ can be decompose into three subspaces and we characterize each one of them. One subspace consists of Riesz fields and another subspace consists of fields of the form cross products of x with a Riesz field. These two subspaces are independent of the Lamé constants.

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