

Diagonal Actions of Groups Acting on Rooted Trees

Dmytro Savchuk

University of South Florida

For a group G acting on a regular rooted d -ary tree T_d and on its boundary ∂T_d we consider the diagonal actions of G on the powers of T_d and ∂T_d . For the action of the full group $\text{Aut}(T_d)$ of automorphisms of T_d on $(\partial T_d)^n$ we describe its ergodic decomposition and compute its Cantor-Bendixson rank. We achieve it with a complete description of the tree of orbits of $\text{Aut}(T_d)$ on T_d^n whose vertices are the orbits of group action on the levels of T_d^n viewed as d^n -ary regular rooted tree. For a subgroup G of $\text{Aut}(T_d)$ the corresponding orbits may be smaller, but sometimes they coincide with the orbits of the full group of automorphisms for all levels. In this case we say that the action of G on $\text{Aut}(T_d)$ is maximally tree n -transitive. For example, maximal tree 1-transitivity is equivalent to level transitivity of the action of G on T_d . We show that the action of Grigorchuk group on ∂T_2 is maximally tree 4-transitive but not maximally 5-transitive. The talk is based on a joint work with Rostislav Grigorchuk and Zoran Šunić.

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