

Algebraic Structure for Constructing Complete Hyperbolic Lattices

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In a pioneering work in the context of communications theory in the hyperbolic plane [1], Brandani considers self-dual tessellations $\{p, p\}$, which form an important subset of the $\{p, q\}$ tessellations. Tessellations of the kind $\{4g, 4g\}$ were considered in [2] in the cases $g = 2, 3$ and in [3] in the cases $g = 2^n, 3 \cdot 2^n, 5 \cdot 2^n$. In both [2] and [3], the arithmetic Fuchsian group formed by the elements that pairing the sides of the polygons obtained by these tessellations are identified in quaternion orders $\mathcal{O}$, which are also called hyperbolic lattices. Furthermore, in [3] denser tessellations than the self-dual $\{4g, 4g\}$ were considered and, therefore, denser lattices were obtained. Hyperbolic lattices $\mathcal{O}$ are the basic entities used in the design of signal constellations in the hyperbolic plane. Once the identification of the arithmetic Fuchsian group in a quaternion order is done, the next step is to present the codewords of the code on a graph, or a constellation of signals (quotient of an order by a proper ideal). However, in order to obtain a complete algebraic labeling, it is necessary that the corresponding order be maximal. An order $\mathcal{M}$ in a quaternion $\mathcal{A}$ is called maximal if $\mathcal{M}$ is not contained in any other order in $\mathcal{A}$, [4]. In [5] the bases of maximal orders on the self-dual tessellations $\{4g, 4g\}$ were presented. The main goal of this work is to describe the maximal orders derived from tessellations denser than $\{4g, 4g\}$, which are an algebraic structure for constructing complete hyperbolic lattices. [1] E. Brandani, Signal Constelations and Performance Analysis on the Hyperbolic Plane. Ph.D dissertation, FEEC-UNICAMP, 2000. [2] E.D. Carvalho, Construction and Labeling of Geometrically Uniform Signal Constellations in Euclidean and Hyperbolic Spaces. Ph.D dissertation, FEEC-UNICAMP, 2001. [3] V.L. Vieira, Arithmetic Fuchsian Groups Identified over the Quaternion Orders for the Construction of Signal Constellations.

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