

Lipschitz continuity of almost minimizers for the p -Bernoulli functional

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In this talk, we consider almost minimizers of the one-phase p -Bernoulli energy functional, which we denote by J_p . The functional J_p is a generalization to each $p > 1$ of the classical one-phase Bernoulli energy functional, where $p=2$, written as J_2 . Almost minimizers of J_2 were investigated in [2,1]. However, D. De Silva and O. Savin provided in [4] a different approach than [2,1], based on nonvariational techniques, to study almost minimizers of J_2 and their free boundaries. Precisely, inspired by [5], they showed that almost minimizers of J_2 are “viscosity solutions” in a more general sense. This property roughly means that almost minimizers satisfy comparison in a neighborhood of a touching point, whose size depends on the properties of the test functions. Once this fact was established, the regularity of the free boundary for almost minimizers followed via the techniques developed by De Silva in [3]. In this talk, we present an optimal Lipschitz continuity result for almost minimizers of J_p with $p > \max(2n/(n+2), 1)$. Our approach is inspired by [4]. Our method mostly relies on using p -harmonic replacements as competitors. The regularity properties of these replacements allow us to infer the Lipschitz continuity of almost minimizers. In particular, we first prove a dichotomy-type result, and next we improve and iterate one of the two alternatives of the dichotomy. The talk is based on joint work with S. Dipierro, F. Ferrari, and E. Valdinoci, see [6]. [1] G. David, M. Engelstein, and T. Toro, Free boundary regularity for almost-minimizers, *Adv. Math.* 350 (2019), 1109-1192 [2] G. David and T. Toro, Regularity of almost minimizers with free boundary, *Calc. Var. Partial Differential Equations* 54 (2015), no. 1, 455-524 [3] D. De Silva, Free boundary regularity for a problem with right hand side, *Interfaces Free Bound.* 13 (2011), no. 2, 223-238 [4] D. De Silva and O. Savin, Almost minimizers of the one-phase free boundary problem,

Comm. Partial Differential Equations 45 (2020), no. 8, 913-930[5] D. De Silva and O. Savin, Quasi-Harnack inequality, Amer. J. Math. 143 (2021), no. 1, 307-331[6] S. Dipierro, F. Ferrari, N. Forcillo, and E. Valdinoci, Lipschitz regularity of almost minimizers in one-phase problems driven by the p -Laplace operator, Indiana Univ. Math. J. 73 (2024), no. 3, 813-854

In Special Session 48 on Free Boundary Problems and Nonlinear PDEs at the Mathematical Congress of the Americas 2025.