Symmetry-Breaking Bifurcations in Networks of Crystal Oscillators for Precision Timing Devices.

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In modern times, satellite-based global positioning and navigation systems, such as the GPS, include precise time-keeping devices, e.g., atomic clocks, which are crucial for navigation and for a wide range of economic and industrial applications. However, precise timing might not be available when the environment renders satellite equipment inoperable. In response to this critical need, we have been carrying out, over the past several years, theory and experiments towards developing a novel and inexpensive precision timing device that can function independently of GPS availability. The fundamental idea is to exploit the symmetry of networks of coupled nonlinear oscillators to generate collective patterns of oscillation that can reduce phase drift. Among all observed patterns, the standard traveling wave, in which consecutive crystals oscillate out of phase by $2\pi/N$, where N is the network size, leads to optimal phase drift error that scales down as $1/N$ as opposed to $1/\sqrt{N}$ for an uncoupled ensemble.

We assume N identical crystal oscillators, where each oscillator is described by a two-mode nonlinear oscillatory circuit. We consider two different topologies, unidirectional and bidirectional, which lead to networks with Gamma=Z_N and Gamma=D_N symmetry, respectively. We apply the method of averaging and show that the full-averaged system is Gamma x O(2) x O(2)-equivariant, where Gamma is the group of symmetries of the network. Then, we present new theoretical results linking symmetry and averaging theory to study the existence and stability of steady-states of the truncated averaged systems, and show that they persist as periodic solutions of the full-averaged system. A decomposition of the phase-space dynamics along irreducible representations of the symmetry groups Z_N and D_N leads to a block diagonalization of the linearized averaged equations.

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Direct computation of eigenvalues leads to the desired identification of periodic solutions that emerge via symmetry-preserving and symmetry-breaking steady-state bifurcations leading to the corresponding periodic solutions with spatio-temporal symmetries.

Then, we provide analytical proof of the scaling laws, for uncoupled and coupled symmetric networks, and show that $1/N$ is the fundamental limit of phase-error reduction that one can obtain with a symmetric network of nonlinear oscillators of any type, not just crystals.

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