The averaging method for Hamiltonian systems in the degenerate case

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In this work we extend the Averaging Theorem for the existence of periodic solutions in Hamiltonian systems or Reeb's Theorem in the degenerate case. More precisely, it is considered the family of analytic Hamiltonian functions$\mathcal{H}_\varepsilon = \mathcal{H}_0(I) + \varepsilon^\alpha \mathcal{H}_1(I, y) + \varepsilon^{\alpha+p}\mathcal{H}_2(I, \theta, y) + O(\varepsilon^{\alpha+p+1})$, $\alpha, p \in \mathbb{N}$, where ε is a small parameter, θ is an angular variable conjugate to the action I. The point y_* is a degenerate (isolated) critical point of $\mathcal{H}_1$ at the fixed level h, i.e., $\mathcal{H}_\varepsilon = h$, $\nabla \mathcal{H}_1(y_*) = 0$ and $Det(Hess\mathcal{H}_1(y_*)) = 0$. If the kernel of $Hess\mathcal{H}_1(y_*)$ (with respect to the variable y) is k with $1 \le k \le 2(n-2)$, then we proved that from the point y_* can bifurcate at most 2^k different periodic solutions of the full system. Furthermore, we provide an approximation of the characteristic multipliers associated with the periodic solutions. We present some illustrative examples of our main results. It can be observed that the maximum number of periodic solutions, represented by 2^k, can be attained.

In Special Session 49 on Bifurcations at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS49-T10