Homeostasis in Input-Output Networks: Structure, Classification and Applications

Fernando Antoneli

Federal University of Sao Paulo (UNIFESP)

Homeostasis occurs in a biological or biochemical system when some output variable remains approximately constant as some input parameters vary over some range. Recently, Golubitsky and Stewart [Homeostasis, Singularities and Networks. J. Math. Biol. 74 (2017) 387-407] introduced the notion of 'infinitesimal homeostasis' allowing the use of implicit differentiation and singularity theory to study homeostasis in systems of ordinary differential equations (ODEs). Generally speaking, ODEs appearing in the life sciences are encoded by 'networks of ODEs'. Nodes correspond to state variables and links indicate which nodes are coupled to which. What distinguishes a network of ODEs from a generic system of differential equations is the capability to keep track of the output from each node individually. Hence, infinitesimal homeostasis is related to occurrence of 'singularities' at individual nodes. In this talk we explain a new approach to the study of the combinatorial structure and classification of homeostasis in 'input-output networks', that is, networks of ODES where we keep track of the output from a fixed node as well as the node(s) that depend on the external input parameters. We employ graph-theoretic ideas from combinatorial matrix theory to provide a systematic way for classifying different types of homeostasis (homeostatic mechanisms) in input-output networks, in terms of the network topology. In turn, this leads to new mathematical concepts, such as, homeostasis subnetworks, homeostasis patterns, homeostasis mode interaction. We illustrate the usefulness of this theory with several biological examples: biochemical networks, chemical reaction networks (CRN), gene regulatory networks (GRN), Intracellular metal ion regulation and so on.

In Special Session 49 on Bifurcations at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS49-T12