Classification of symmetric chaos

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Symmetric dynamical systems can show complex phenomena that only reveal their symmetries (either the full symmetry group of the system, a smaller subgroup, or no symmetry) when considering long-time trajectories. Identifying the subgroup that characterizes the symmetry of the attractor, is relevant for situations where the structure of the chaotic attractor suffers qualitative changes, as in crises and other bifurcations. Although there are well-established methods for the classification, most notably the 'symmetry detectives' developed by Barany, Dellnitz & Golubitsky (1993), recent advances in machine learning and data-driven modeling offer promising new approaches to this problem.In this presentation, I will show two new ways of addressing the issue that benefit from recently developed theory and rely on computer algebra software.The first method is based on Optimal Transport and can efficiently quantify the distance between an invariant measure (either a long sequence of points or a multi-dimensional histogram) and its transformations under the actions of the group, and assess the presence of a given subgroup. We show the potential of the new method using systems of coupled oscillators that are either fully symmetric or composed of symmetric subpopulations.The second method is based on the Perron-Frobenius operator (the Koopman operator can also be used) and reveals the presence of multiple conjugate attractors.

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