

Extending twist construction for Modal Nelson lattices

María Paula Menchón

Nicolaus Copernicus University in Toruń / CONICET (Argentina)

In this study, we explore the extension of Nelson residuated lattices (N3) with a unary modal operator. We introduce a variety of modal Nelson lattices and demonstrate that they can be characterized by twist structures. We followed this line of research and work over one of the most challenging research trends in non-classical logic: the attempt to combine different non-classical approaches, in our case Nelson logic and modal logic. This combination offers the skill of dealing with modal notions like belief, knowledge, and obligations in interaction with other aspects of reasoning that can be best handled using many-valued logics, for instance, vagueness, incompleteness, and uncertainty. The study that we are going to introduce could be especially interesting from the point of view of Theoretical Computer Science and Artificial Intelligence. To achieve this, we first extend the well-known twist construction of Nelson lattices to the modal setting. . Every Nelson lattice can be represented as a twist-structure over a Heyting algebra. A Twist structure over a lattice is a construction used by Kalman in [2] that allows us to represent an algebra as a subalgebra of a special binary power of the lattice which is obtained by considering its direct product and its order-dual. This construction has gained increasing importance in recent years in the study of algebras related to non-classical logics. Our proposed extension is more general than previous approaches in the literature, as it does not require monotonicity with respect to modal operators. We introduce the variety of modal Heyting algebras and show that the categories of modal Nelson and modal Heyting algebras with a Boolean filter are equivalent. The underlying logic of our variety of modal Heyting algebras corresponds to the Hilbert system IE3 from [1, Section 4.2]. References[1] Dalmonete, C. Grellois, and N. Olivetti. Intuitionistic Non-normal Modal

Logics: A General Framework. *Journal of Philosophical Logic* 49, 833–882 (2020). <https://doi.org/10.1007/s10992-019-09539-3>[2] J. A. Kalman. Lattices with involution, *Trans. Amer. Math. Soc.*, vol. 87 (1958), pp. 485–491.

In Special Session 50 on Algebraic Logic at the Mathematical Congress of the Americas 2025.