Cutting of random recursive trees

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Increasing trees are rooted trees, where each vertex has a unique label and the labels along paths away from the root are in increasing order. A Random recursive tree on n vertices (abbreviated as RRTs) is a tree chosen uniformly at random from the set of increasing trees with vertex labels $\{1, \ldots, n\}$. The idea of cutting random recursive trees was introduced by Meir and Moon in 1974. They studied the following procedure: Start with a random recursive tree on n vertices. Choose an edge at random and remove it, along with the cut subtree that does not contain the root. Repeat until the remaining tree consists only of the root; at which point, we say that the tree has been deleted. Let X be the number of edge removals needed to delete a RRT with n vertices. The random variable X has been thoroughly studied and analogous variables under distinct models of random trees have been analyzed; in particular, X grows asymptotically as $n \ln(n)$. In this talk we propose and study two methods for cutting down a random recursive tree. The first one, the targeted vertex-cutting, focuses on its largest degree vertices. Enumerate the vertices of a random recursive tree of size n according to a decreasing order of their degrees. The process is performed by sequentially removing vertices according to that order and keeping only the subtree containing the root after each removal. The algorithm ends when the root is picked to be removed. The second one, the degree-biased cutting, is similar to the Meir and Moon procedure, but now vertices are deleted with probability proportional to their degree. For both methods we present upperbounds of the order of growing of X, the number of removals needed to delete the RRT.

In Special Session 51 on Discrete Stochastic Models and Applications at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS51-T10