Tamer regularity lemmas and PAC learning theory

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One of the most influential tools in extremal combinatorics is the Szemerédi Regularity Lemma. In its base form, this lemma states that sufficiently large graphs can partitioned into few parts and edge densities between (most of these) these parts are close to "quasirandom". In other words, large graphs can be well-approximated by smaller "reduced graphs" with weighted edges. Recently, we have seen the emergence of a sub-field called "tamer regularity lemmas", whose main idea is to obtain improvements on regularity lemmas when we restrict the graphs to be of a hereditary class with a particular property, and more generally, characterize which classes admit such improvements. One prominent example is the ultra strong regularity lemma, which says that a hereditary class admits a regularity lemma with polynomial bounds if and only if it admits a regularity lemma with a 0/1-valued reduced graph if and only if the class omits some bipartite pattern. The phenomenon that characterizes the ultra strong regularity lemma has actually appeared under different names in other fields of mathematics. It is known as NIP (standing for "not independence property") in model theory and as finite Vapnik–Chervonenkis (VC) dimension in learning theory. Using ideas of model theory, logicians were able to obtain satisfactory hypergraph analogues of the 0/1-valued regularity lemmas and more recently, drawing from ideas of learning theory (more specifically, the Haussler packing property), they were also able to obtain hypergraph regularity lemmas with improved bounds for well-behaved classes. However, this latter connection to PAC learning theory is still weak, essentially because classic PAC learning theory is missing a high-arity analogue. In this talk, I will briefly review these tamer regularity lemmas and present our recently developed theory of high-arity PAC learning theory, which may connect better to hypergraph regularity

MCA 2025 internal code: SS52-T03

lemmas than its classic counterpart. No background in regularity lemmas, model theory or learning theory will be required. This talk is based on joint work with Maryanthe Malliaris.

In Special Session 52 on Extremal and Probabilistic Combinatorics at the Mathematical Congress of the Americas 2025.