

Arbitrary spectral edge of regular graphs

Dingding Dong

Harvard University

We prove that for each $d \geq 3$ and $k \geq 2$, the set of limit points of the first k eigenvalues of sequences of d -regular graphs is $\{(\mu_1, \dots, \mu_k) : d = \mu_1 \geq \dots \geq \mu_k \geq 2(d-1)^{(1/2)}\}$. The result for $k = 2$ was obtained by Alon and Wei, and our result confirms a conjecture of theirs. Our proof uses an infinite random graph sampled from a distribution that generalizes the random regular graph distribution. To control the spectral behavior of this infinite object, we show that Huang and Yau's proof of Friedman's theorem bounding the second eigenvalue of a random regular graph generalizes to this model. We also bound the trace of the non-backtracking operator, as was done in Bordenave's separate proof of Friedman's theorem.

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