

On some hypergraph coloring problems

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Of a given bipartite graph $G = (V, E)$, it is elementary to construct a bipartition in time $O(|V|+|E|)$. For a given k -graph $H = H(k)$ with fixed k at least 3, Lovasz proved that it is NP-complete to decide whether H is bipartite. Let $B_n = B_n(k)$ denote the collection of $[n]$ -vertex bipartite k -graphs. We construct, of a given H of B_n , a bipartition in time averaging $O(nk)$ over the class B_n . When $k = 3$, this result expedites one of Person and Schacht. We also consider the class $F_n = F_n(3)$ of $[n]$ -vertex 3-graphs $H = H(3)$ containing no copies of the Fano plane $F = F(3)$ as a subhypergraph. We construct, of a given H of F_n , a smallest proper coloring in time averaging $O(n^3)$ over the class F_n . This result also expedites one of Person and Schacht.

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