56 - The toric Euler-Jacobi vanishing theorem and its converse

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The classical multivariate Euler-Jacobi theorem asserts that given n complex polynomials in n variables having d common isolated roots in $\mathbb{C}^n$, where d equals the product of their degrees, the sum of local residues at all these roots vanishes for any polynomial with degree smaller than the critical degree defined as the sum of the degrees, minus n. This sum of local residues defines a global object in projective space and its vanishing is indeed equivalent to the fact that there are no zeros at infinity. In the toric case, the input is a system of sparse Laurent polynomials with fixed Newton polytopes and the first version of the toric Euler-Jacobi vanishing theorem is due to Khovanskii, under some genericity assumptions. In joint work with Carlos D'Andrea we give conditions on the polytopes to ensure that this global vanishing is equivalent to the existence of zeros at infinity in the associated toric variety. We relate these conditions with the dimension of the quotient in the toric critical degree of its homogeneous coordinate ring by the ideal generated by the multihomogenizations of the input polynomials. We also relate the existence of zeros at infinity with interpolation questions.er your abstract here.

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