

D-extremal ideals

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Extremal ideals are a class of square-free monomial ideals which bound many homological invariants, and determine much of the algebraic behaviour, of all square-free monomial ideals and their powers. More specifically, if q is a positive integer, then the q -extremal ideal $\mathcal{E}_q$ has the property that for every $r > 0$, the power $\mathcal{E}_q^r$ determines or bounds the algebraic behaviour of I^r when I is any square-free monomial ideal with q generators. The generators of q -extremal ideals have no relations between them. In other words no generator of $\mathcal{E}_q$ divides the least common multiple of other generators. In that sense, extremal ideals have the largest number of minimal syzygies among all the square-free monomial ideals they represent, and this feature is what leads them to bound the betti numbers of all square-free monomial ideals (and their powers). A natural question is if, fixing a positive integer q , one could find an ideal which models and bounds, more effectively, the behaviour of *all* square-free monomial ideals with q generators which have divisibility relations among them. For example, ideals such that the first generator divides the lcm of the second, third and fourth generators. From the integer q and a set $\mathcal{D}$ of divisibility relations such as the one described above, we build the $\mathcal{D}$ -extremal ideals, which shows algebraically extremal behaviour among all square-free monomial ideals satisfying those divisibility relations. In this talk we will introduce the class $\mathcal{D}$ -extremal ideals and demonstrate how they provide much tighter bounds than $\mathcal{E}_q$ on homological invariants for all powers of any square-free monomial ideal whose generators satisfy the the divisibility relations in $\mathcal{D}$. This talk is based on joint work with Susan M. Cooper, Sabine El Khoury, Susan Morey, Liana M. Sega, and Sandra Spiroff.

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