Ischebeck's Formula, Grade and Quasi-homological Dimensions

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The quasi-projective dimension and quasi-injective dimension are recently introduced homological invariants that generalize the classical notions of projective dimension and injective dimension, respectively. For a local ring R and finitely generated R-modules M and N, we present conditions involving quasi-homological dimensions where the equality $\sup\{i \geq 0 : \operatorname{Ext}^i_R(M,N) \neq 0\} = \operatorname{depth} R - \operatorname{depth} M$, which we call Ischebeck's formula, holds. On the other hand, we establish an inequality relating the quasi-projective dimension of a finitely generated module to its grade and introduce the concept of a quasi-perfect module as a natural generalization of a perfect module. Additionally, we show a formula for the grade of finitely generated modules with finite quasi-injective dimension over a local ring. This is a joint work with Victor H. Jorge Pérez and Paulo Martins.

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