

Outer Symplectic Billiards

Ana Cristina Chávez Cáliz

Mathematical Institute of Heidelberg University, Germany

In 1959, B. Neumann introduced outer billiards in the plane, a discrete-time dynamical system defined in the exterior of an oriented plane oval. The outer billiard maps a point z to another point z' if the line zz' is tangent to the curve at the midpoint $Q=(z+z')/2$. This map preserves area, and, using KAM theory, J. Moser showed that if the curve is smooth enough, the orbits of this billiard do not escape to infinity. Outer symplectic billiards generalize outer billiards to higher-dimensional symplectic spaces. Using a variational approach, we can establish the existence of odd-periodic orbits. Interestingly, however, we cannot guarantee the existence of even-periodic orbits. We will also discuss the behavior of this correspondence when the "table" is a curve or a Lagrangian submanifold. This is joint work with P. Albers and S. Tabachnikov.

In Special Session 57 on Differential Equations and Geometric Structures at the Mathematical Congress of the Americas 2025.