

Asymptotic lines and curvature lines of surfaces

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Each surface lying in the 3-dimensional Euclidean space has two sets of foliations, possibly with singularities, defined by the geometry inherited by the surface from the ambient space, that is, its extrinsic geometry. These foliations are called the asymptotic lines and the curvature lines of the surface. Their study has appealed to the interest of many mathematicians since the nineteenth century. They represent a spot where geometry, topology, and the theory of differential equations converge. In this talk, we describe the global foliation of asymptotic lines on compact surfaces of genus greater than one, defined in $S^3 \subset \mathbb{R}^4$ as the links of the real part of the Milnor fibration of a family of polynomials. Through this analysis, we present a description of the global foliation of curvature lines of the embeddings, defined as the images of the stereographic projection of these links into $\mathbb{R}^3$. Since G. Monge described the case of the triaxial ellipsoid by direct integration of the differential equation of these lines, [1], [2] A complete analytic description of them has not been achieved for the case of a compact orientable surface of genus greater than one. We provide the first embeddings of compact surfaces of genus greater than one in $\mathbb{R}^3$ whose curvature lines have a global description [3], see also [4]. This is a joint work with Vinicio Gómez-Gutiérrez. Each model we describe has only four singularities, called umbilic points, which are not generic for all cases of genus greater than 2. The study of bifurcations of these umbilic points is an interesting subject to develop using the tools of our approach. Acknowledgements Supported by Grant PAPIIT-IT102924-DGAPA-UNAM

References

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