Projection Mappings and the Gradient Projection Method on Hyperbolic Space Forms

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This paper investigates the gradient projection method for solving constrained optimization problems in κ-hyperbolic space forms, which may involve non-convex objective functions. We analyze both constant and backtracking step sizes, focusing on the hyperboloid model (Lorentz model). We introduce several properties of the intrinsic κ-projection onto convex sets in κ-hyperbolic space forms, discussing their relevance to the method's analysis. We further examine the relationship between intrinsic κ-projections, Euclidean orthogonal projections, and Lorentz projections, providing formulas for specific convex sets. Convergence results demonstrate that each accumulation point generated by the method with backtracking step sizes is a stationary point. With Lipschitz-continuous gradients, the method with a constant step size also ensures stationary points. We provide iteration complexity bounds for achieving stationarity with both step sizes. Lastly, we discuss the constrained Fermat-Weber problem, showing that the gradient projection method converges to its unique solution.

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