

Barycenter and k -barycenter estimation with unknown distances

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We will consider the consistency of the k -(Fréchet) means in metric spaces when the distances are unknown and have to be estimated. We will show that the empirical k -means are consistent as far as the empirical metric spaces converge to the population one in measured Gromov-Hausdorff sense. As a consequence we obtain (a) consistency of k -means computed with Fermat distances, (b) consistency of k -means computed with Isomap geodesic distance estimator and (c) consistency of empirical k -Wasserstein barycenter computed with sample based estimators of the measures involved. For (a) and (b) these results are new even for the case $k = 1$, as far as we know. For (c) they are well-known for $k = 1$ but not for $k > 1$ up to our knowledge. We do not assume the uniqueness of the barycenter, but when the barycenter is in fact unique, the results are stronger.

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