Stability of tetrahedral frame-field ground states for a tensor-valued relaxed variational model

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We consider a variational problem for tensor-valued functions to describe frame-valued fields. Frame fields have applications to some classes of nematic liquid crystals and frustrated media. By a frame, we refer to a set of m vectors in R^n that satisfy some symmetry conditions. Of particular interest is a set of n orthogonal lines in R^n, known as an n-cross. Recently, Golovaty et al. introduced a new framework, [1], that applies in arbitrary dimension n>=2 that associates an n-cross with a symmetric 4-tensor, that satisfies some trace conditions and a nonlinear constraint. By relaxing such a constraint, the authors derive a Ginzburg-Landau type functional for relaxed, tensor-valued maps. More recently, in [2], Golovaty et al. manage to adapt a new selection principle for the optimal generation of tetrahedral frame fields (n=3) in R^3 in Lipschitz domains. The novel feature of their variational relaxation approach is the recovery procedure that allows for the extraction of the tetrahedral frame from a constrained 3-tensor. We analyze the linear stability properties of the tetrahedral ground states of this Ginzburg-Landau type energy. The proof is mainly based on Fourier-analysis techniques, originally used by Mironescu in [3] to analyze the linear stability of the degree-one vortex ground state of the Ginzburg-Landau energy, and more recently by Lamy & Zuniga to obtain a similar result for such a vortex in the context of the anisotropic Ginzburg-Landau energy, [4]. This work is a collaboration with Dr. Daniel Spirn (School of Mathematics & Insitute for Mathematics and Applications, University of Minnesota, MN, USA).

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References

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