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# Legendrian links and their Lagrangian fillings

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We will discuss the geometric problem of classifying Lagrangian surfaces in the 4-ball that bound Legendrian links in the boundary 3-sphere, up to Hamiltonian isotopy. The talk will use motivating examples and known results as guiding principles, and introduce the key ingredients and constructions from the ground up. In the course of exploring this problem 4-dimensional problem, we will see how cluster algebras and quivers naturally appear, providing a fruitful connection between symplectic topology, representation theory and algebraic combinatorics. The main new results will be of geometric nature but, interestingly, these algebraic aspects are needed in crucial parts of the argument. The talk will conclude with a precise conjecture on the classification of these Lagrangian surfaces, and provide a breakdown of what is known, what is work-in-progress, and potential future efforts.